Ion traps

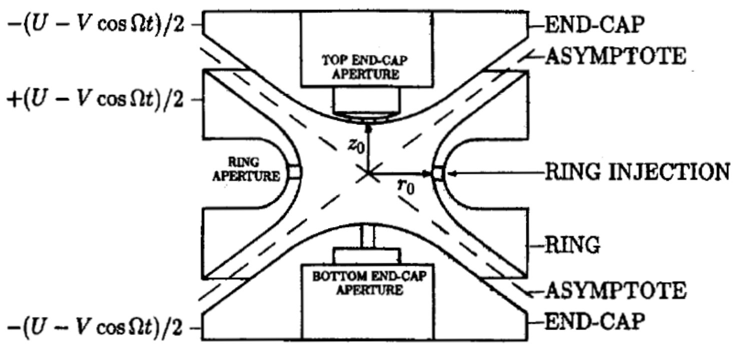
Note Title 2/5/2005

Electric traps - charged atoms (ions)

rf Paul trap (there are also Penning traps, which have DC electric and magnetic fields)

- laser cooling 1st demonstrated in ion trap
- best electron g-2 measurement for e⁻ in Penning trap
- leading quantum computing experiments done in ion traps
- best mass spectroscopy in ion traps
- possible future frequency standards in ion traps

Classic quadupole Paul trap:



if the surfaces are perfect hyperboloids, with $r_0^2 = 2z_0^2$, then the electric potential is:

$$\phi = \phi_0 \frac{x^2 + y^2 - 2z^2}{2r_0^2} \cos \Omega t$$

which gives classical equations of motion: (assuming singly-charged positive ion)

$$\left.\begin{aligned} \ddot{r} + \frac{e}{mr_0^2}(U - V\cos\Omega t)\, r = 0 \\ \ddot{z} - \frac{2e}{mr_0^2}(U - V\cos\Omega t)\, r = 0 \end{aligned}\right\} \text{Mathieu equations}$$

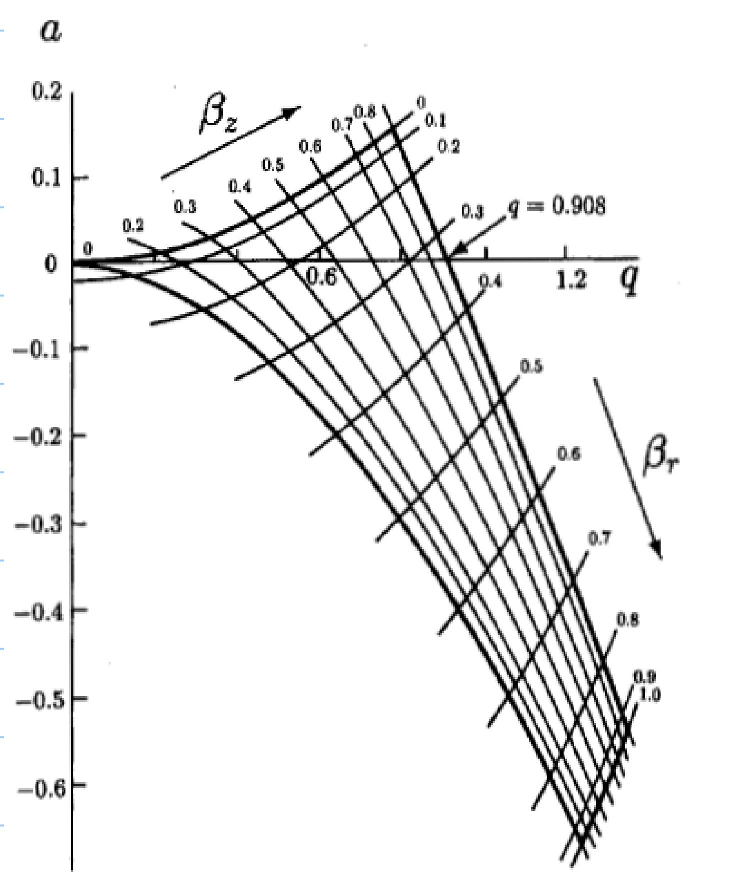
define: $a_z = -2a_r = -\dfrac{8eU}{mr_0^2\Omega^2}$ $\quad q_z = -2q_r = -\dfrac{4eV}{mr_0^2\Omega^2}$

$\zeta = \Omega t / 2$

$$\left.\begin{aligned} \frac{d^2r}{d\zeta^2} + (a_r - 2q_r\cos 2\zeta)\, r = 0 \\ \frac{d^2z}{d\zeta^2} + (a_z - 2q_z\cos 2\zeta)\, z = 0 \end{aligned}\right\}$$

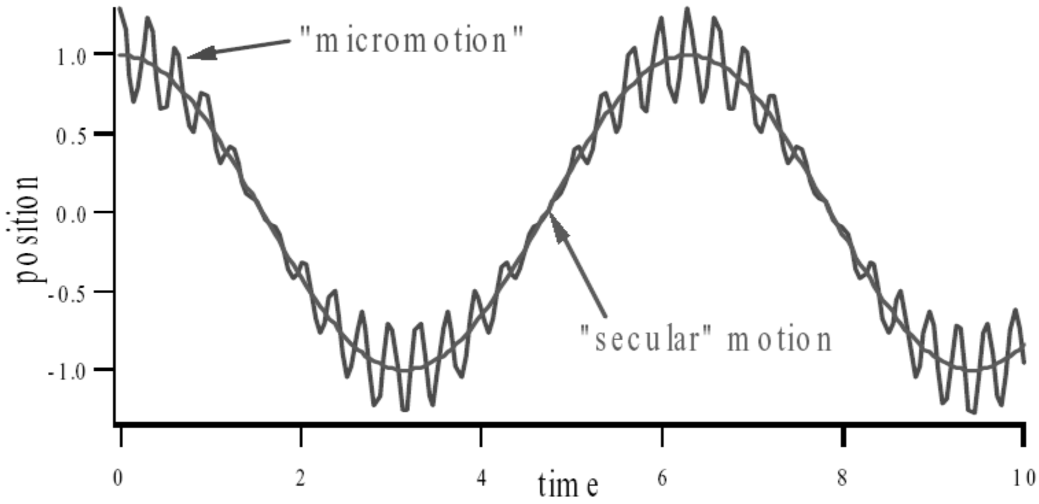
There are exact solutions to these equations, which have regions of stability —

- the ion is confined not really by an averaged force, but by a ponderomotive force
- the time-dependent saddle keeps "kicking" the ion back to the middle — this exquisite timing requires some constraints!



by this principle — ion traps used as e/m filters in mass spectroscopy!

Motion looks like this in 1-D:



The pseudopotential approximation is useful:

write $\boxed{z = Z + \delta}$

secular motion - low frequency

micromotion - high frequency

and assume $\delta \ll Z$ and $\dot{\delta} \gg \dot{Z}$

The Mathieu equation in z becomes:

$$\frac{d^2\delta}{d\zeta^2} = -(a_z - 2q_z\cos 2\zeta)Z$$

if Z is roughly constant over one period of δ, and $a_z \ll q_z$, then integrate to get:

$$\delta = -\frac{q_z Z}{2}\cos 2\zeta$$

now $z = Z + \delta = Z - \frac{q_z Z}{2}\cos 2\zeta$ and the Mathieu equation for z is:

$$\frac{d^2 z}{d\zeta^2} = \frac{d^2}{d\zeta^2}(Z+\delta) = -a_z Z + \frac{q_z q_z}{2} Z\cos 2\zeta + 2q_z Z\cos 2\zeta - q_z^2 Z\cos^2 2\zeta$$

over one period of the rf, $\left\langle \frac{d^2\delta}{d\zeta^2} \right\rangle = 0$

So, $\left\langle \frac{d^2 Z}{d\zeta^2} \right\rangle = \frac{1}{\pi}\int_0^{\pi} \frac{d^2 z}{d\zeta^2}\, d\zeta$

$$= -\left(a_z + \frac{q_z^2}{2}\right) Z$$

$$\left\langle \frac{d^2 Z}{dt^2} \right\rangle = -\left(a_z + \frac{q_z^2}{2}\right) \frac{\Omega^2}{4} Z$$

$$\beta_z \equiv \sqrt{a_z + \frac{q_z^2}{2}} \qquad \omega_z \equiv \beta_z \frac{\Omega}{2}$$

$$\left\langle \frac{d^2 Z}{dt^2} \right\rangle = -\frac{\beta_z^2 \Omega^2 Z}{4} = -\omega_z^2 Z$$

w/ no DC bias, $a_z = 0$ and $q_z = \frac{2eV}{m z_0^2 \Omega^2}$

$$\boxed{\left\langle \frac{d^2 Z}{dt^2} \right\rangle = -\frac{e^2 V^2}{2 m^2 z_0^4 \Omega^2} Z}$$

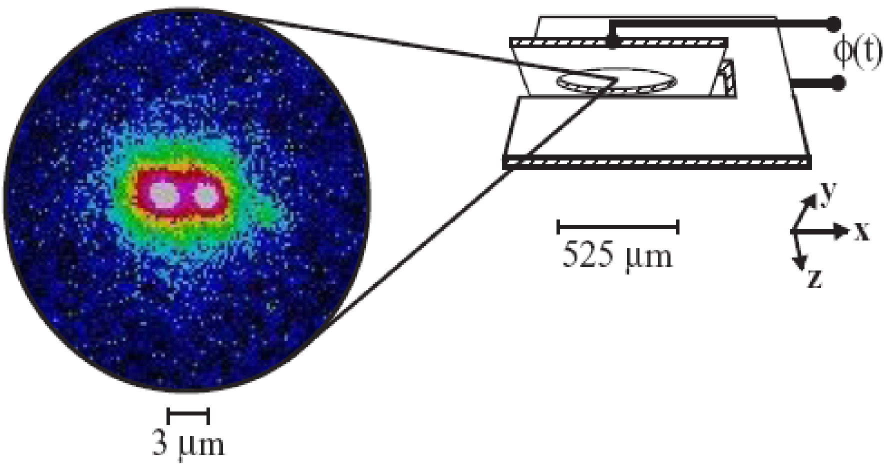
This is just a harmonic oscillator, generated by a "pseudopotential"

$$D_z = \frac{m z_0^2 \Omega^2 q_z^2}{16 e}$$

The same trick works in the radial direction

quantum treatment: complicated, but the end result is a 3-D harmonic oscillator with states that "breathe" at Ω

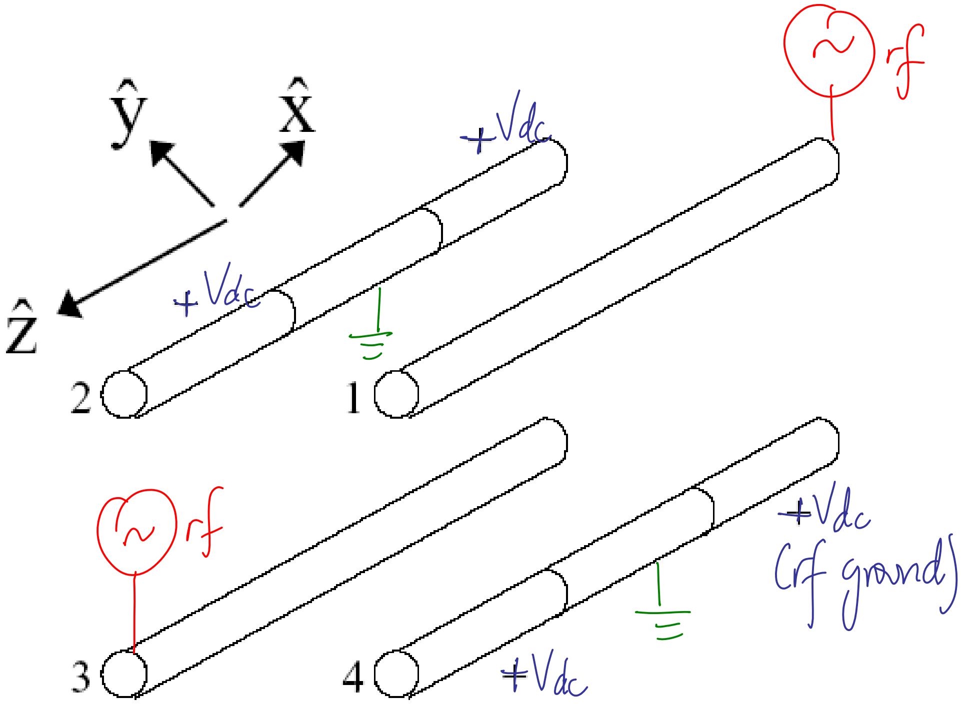
Modern version: "ring and fork" trap

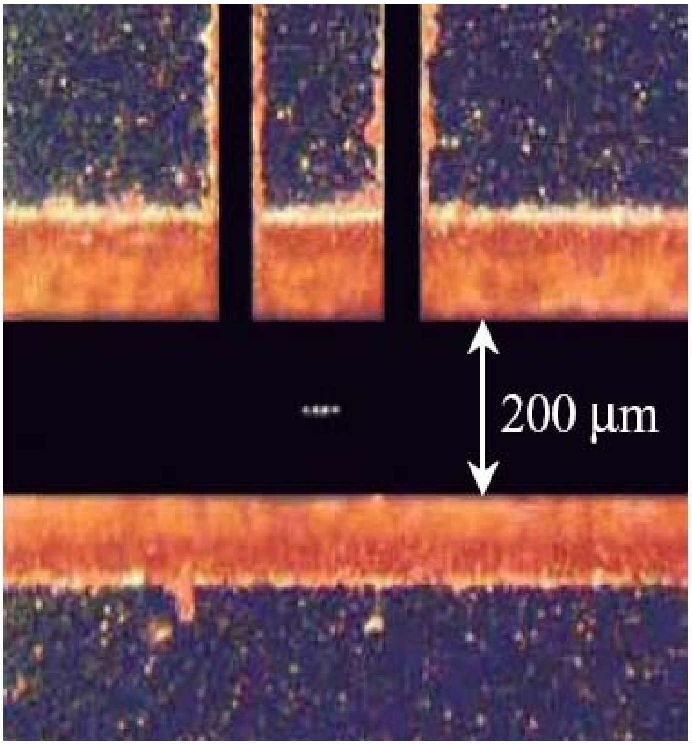


these types of traps are very anharmonic away from the center

Note! δ is only 0 at $z_i = 0$, so with more than one ion there is always micromotion!

A way around this: linear Paul trap





w/ $^9Be^+$

$V_{dc} \sim 10V$

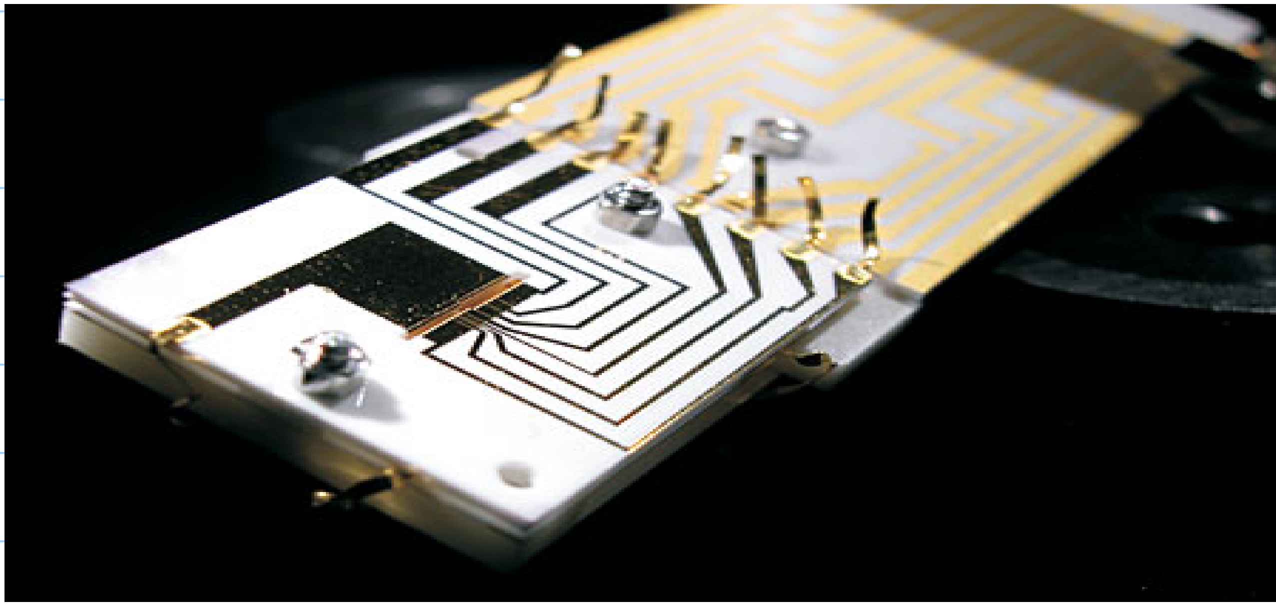
$V_{rf} \sim 500V$

$\Omega \sim 100$ MHz

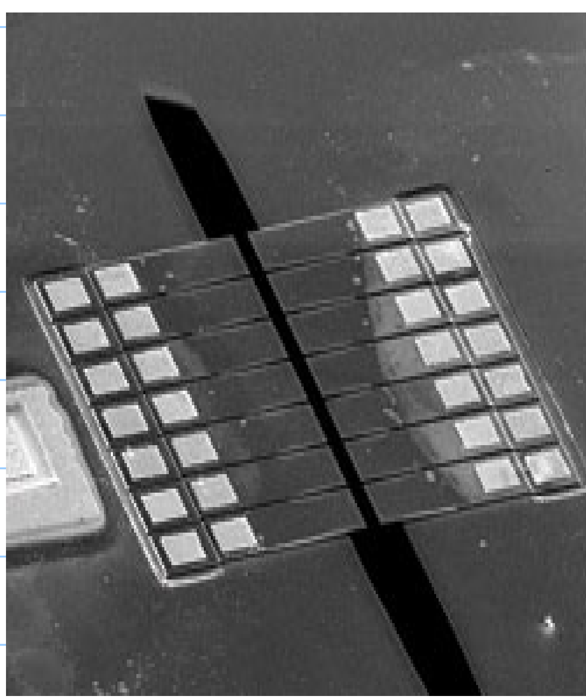
$\omega_r \sim 2\pi\, 3$ MHz

$\omega_z \sim 2\pi\, 1$ MHz

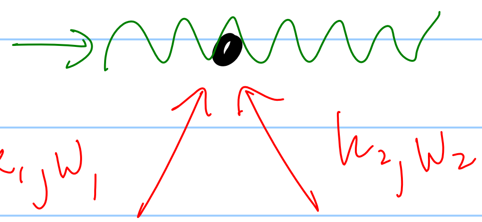
Micromachined trap :



Lithographic traps



Because such strong confinement is possible in ion traps, they can operate in the resolved sideband limit and can be cooled using stimulated Raman transitions into the 3D motional ground state.



if $k_1 \neq k_2$, then SR transitions can be used to change the atomic motion through recoil — one way to think about this is to consider the AC Stark shift giving rise to a moving periodic potential that can exert a force on the atom

See the auxiliary SR notes for details on this in free space...

Life is more complicated for a harmonic oscillator (like a Paul trap)

$$H_0 = \hbar\omega |n\rangle\langle n| + \hbar\omega_0 |g_2\rangle\langle g_2| + \hbar\omega_{eg} |e\rangle\langle e|$$

$$|\psi\rangle = \sum_n \left(c_{1n}|g_1\rangle + c_{2n}|g_2\rangle + c_{en}|e\rangle \right) |n\rangle$$

Similar analysis, but $|n\rangle$ are not momentum eigenstates

the result:

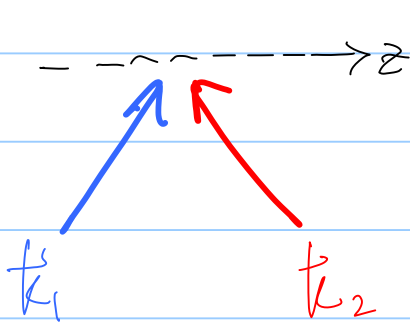
transitions $|g_1\rangle|n\rangle \longleftrightarrow |g_2\rangle|n'\rangle$ are possible

with Rabi rate: $\Omega = \frac{\Omega_1 \Omega_2^*}{\Delta} \langle n'| e^{i \Delta\vec{k}\cdot\vec{x}} |n\rangle$

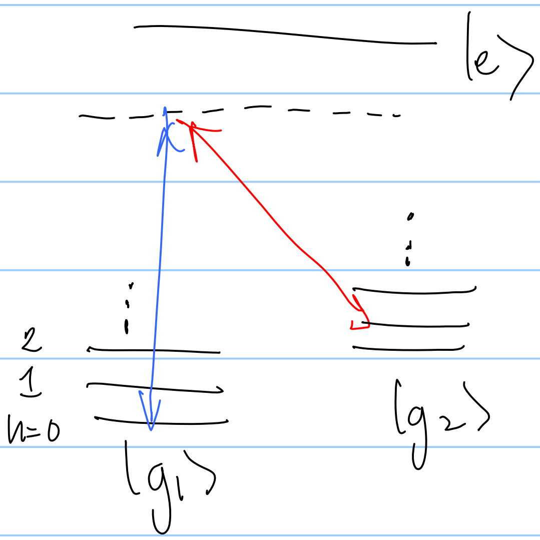
$$= \frac{\Omega_1 \Omega_2^*}{\Delta} \langle n'| e^{i\eta(a+a^\dagger)} |n\rangle$$

where $\eta = \Delta k_z \cdot z_0$ is the Lamb-Dicke parameter

$\left(z_0 = \sqrt{\frac{\hbar}{2m\omega_z}} \right.$ is the harmonic oscillator length$\left. \right)$



$\Delta \vec{k} = \vec{k}_1 - \vec{k}_2$ wave vector difference



In the Lamb-Dicke limit, as $\Delta n = n' - n$ gets bigger, the Rabi rate is strongly suppressed

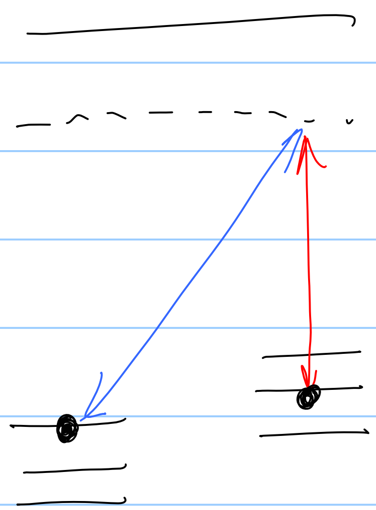
$$\langle n' | e^{i \Delta k_z z} | n \rangle = e^{-\eta^2/2} \sqrt{\frac{n_<!}{n_>!}}\, \eta^{|n'-n|}\, \underbrace{L_{n_<}^{|n'-n|}(\eta^2)}_{\text{generalized Laguerre polynomial}}$$

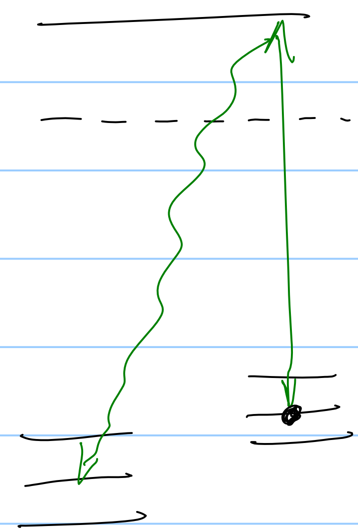
extremely similar to what is seen in x-ray Bragg scattering – "Debye-Waller factor"

Stimulated Raman cooling:

We can cool an ion to $T=0$!

Multi-stage cooling process:





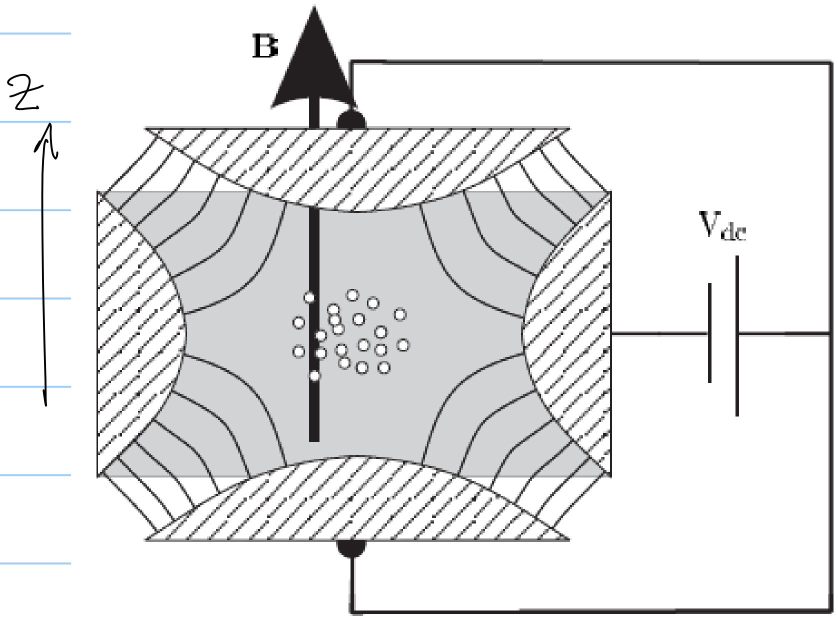
① π-pulse $|g_1, n\rangle \rightarrow |g_2, n-1\rangle$

② pump back using spontaneous emission — in the LD limit:

$|g_2, n-1\rangle \rightarrow |g_1, n-1\rangle$

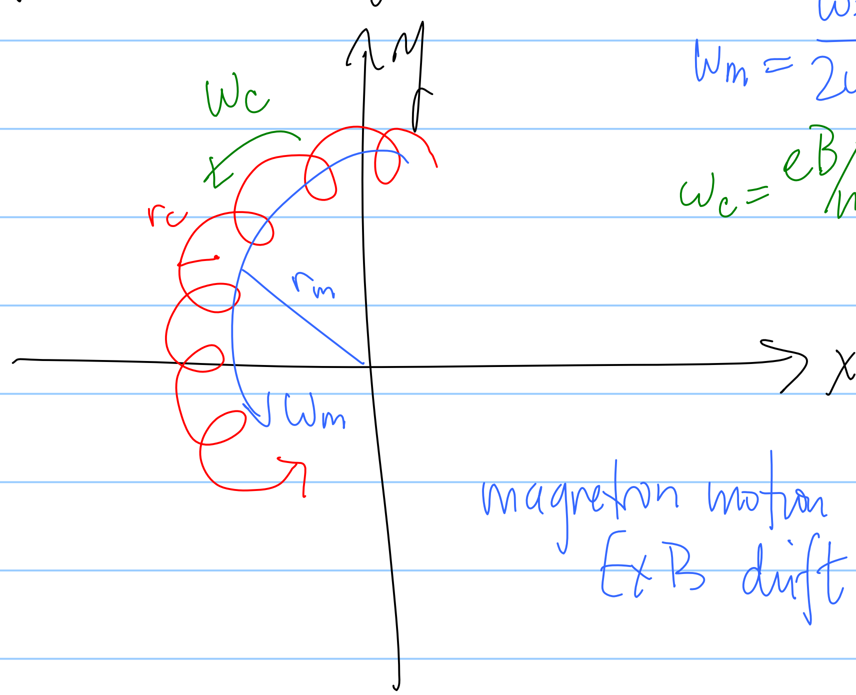
③ repeat!

Penning traps



Paul trap with static voltages and a uniform magnetic field

The equations of motion for an ion now include the Lorentz force $\propto \vec{v} \times \vec{B}$, and we get trajectories like:

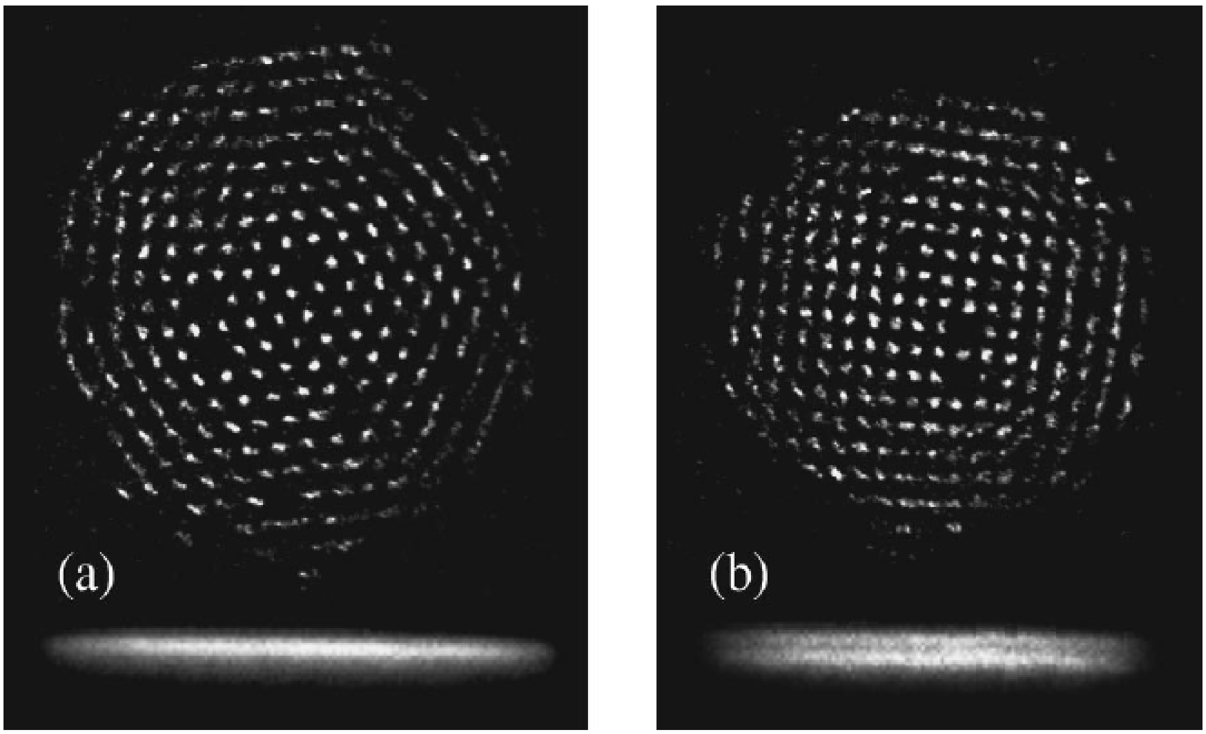


$\omega_m = \frac{\omega_z^2}{2\omega_c}$ magnetron frequency

$\omega_c = eB/m$ cyclotron frequency

magnetron motion is caused by $E \times B$ drift!

Confinement along the z-direction from the endcaps is harmonic... so, three, independent harmonic degrees of freedom, with cyclotron radius r_c and magnetron radius r_m.



<u>A lot</u> of ions can be trapped in a Penning trap!